Clocks Calender and Age

Important Formula and Equations

Minute Spaces: The face or dial of watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

Hour Hand and Minute Hand: A clock has two hands, the smaller one is called the hour hand or short hand while the larger one is called minute hand or long hand .

In 60 minutes, the minute hand gains 55 minutes on the hour on the hour hand.

In every hour, both the hands coincide once.

The hands are in the same straight line when they are coincident or opposite to each other.

When the two hands are at right angles, they are 15 minute spaces apart.

When the hands are in opposite directions, they are 30 minute spaces apart.

Angle traced by hour hand in 12 hrs = 360°

Angle traced by minute hand in 60 min. = 360°.

If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast .

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow .

Odd Days: We are supposed to find the day of the week on a given date. For this, we use the concept of 'odd days'. In a given period, the number of days more than the complete weeks are called odd days .

Leap Year: (i). Every year divisible by 4 is a leap year, if it is not a century. (ii). Every 4th century is a leap year and no other century is a leap year.

Ordinary Year: The year which is not a leap year is called an ordinary years . An ordinary year has 365 days.

Counting of Odd Days: 1 ordinary year = 365 days = (52 weeks + 1 day.) . 1 ordinary year has 1 odd day.

$\qquad$ 1 leap year = 366 days = (52 weeks + 2 days) 1 leap year has 2 odd days.
$\qquad$ 100 years = 76 ordinary years + 24 leap years
$\qquad\qquad$ = (76 x 1 + 24 x 2) odd days = 124 odd days.
$\qquad\qquad$ = (17 weeks + days) = 5 odd days.

Number of odd days in 100 years = 5

Number of odd days in 200 years = (5 x 2)= 3 odd days.

Number of odd days in 300 years = (5 x 3) = 1 odd day.

Number of odd days in 400 years = (5 x 4 + 1) = 0 odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.

Day of the Week Related to Odd Days (Assuming that 1AD January 1st is a Sunday):

No. of days:	0	1	2	3	4	5	6
Day:		Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

Key Notes:

Clocks Concepts :

The dial of the clock is circular in shape and was divided into 60 equal minute spaces

60 minute spaces traces an angle of 360°. Therefore, 1minute space traverses an angle of 6°

In 1 hour, Minute hand traverses 60 minute space or 360° ,Hour hand traverses 5 minute space or 30°

The hands of the clock are perpendicular in 15 minute spaces apart

The hands of the clock are in straight line and opposite to each other in 30 minute spaces apart.

The hands of the clock are in straight line when they coincide or opposite to each other.

The hands of the clock are perpendicular to each other for 22 times in 12 hours and for 44 times in a day.

The hands of the clock are opposite to each other for 11 times in 12 hours and 22 times in a day.

The hands of the clock coincides with each other for 11 times in 12 hours and 22 times per day

The hands of the clock are 44 times in a straight line per day

The minute hand gain 55 minutes over hour hand per hour.

Hence x minute space to be gained by minute hand over hour hand can be calculated as x.(60/55) or x.(12/11)

Ex : At what time between 2'O clock and 3'O clock the hands of the clock
are opposite to each other.
1. 34(6/11) past 2'Oclock 2. 43(7/11) past 2'Oclock
3. 56(8/11) past 2'Oclock 4. 64(9/11past 2'Oclock
Sol At 2'O clock the minute hand will be at 12 as shown below
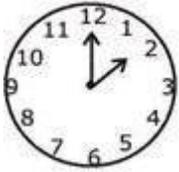
The minutes hand to coincide with the hour hand it should trace at first 10 minute spaces
And then the hands of the clocks to be opposite to each other minute hand should trace 30 minute spaces i.e.
totally it should gain 10+30=40 minute spaces to be opposite to that of hour hand
We know that,
Minute hand gains 55 minute spaces over hour hand in 1 hour
Therefore, Minute hand gain 40 minute spaces over hour hand in 40 × (60/55) = 43(7/11)
Hence the hand of the clock will minutes be opposite to each at 43(7/11) past 2'Oclock
Therefore, Correct option is 2'

When clock is too fast, too slow
If a clock or watch indicates 6 hr 10 min when the correct time is 6, it is said that the clock is 10 min too fast
If it indicates 6. 40 when the correct time is 7, it is said to be
20 min too slow.
Now let us have an example based on this concept

Ex. My watch, which gains uniformly, is 2 min, & show at noon on Sunday, and is 4 min 48 seconds fast at 2 p.m on the following Sunday when was it correct ?

Sol: From Sunday noon to the following Sunday at 2 p.m there are 7 days 2 hours or 170 hours.
The watch gains 2+4 $^4/_5$ min in 170 hrs.
Therefore, the watch gains 2 min in $\underline{2}$ *170 hrs i.e., 50 hours
$6 ^4/_5$
Now 50 hours = 2 days 2 hrs.
Therefore, 2 days 2 hours from Sunday noon = 2 p.m on Tuesday.

Calendars Concept :

The time in which the earth travels round the sun is a solar year and is equal to 365 days 5 hrs. 48 minutes and 47 $^1/_2$ seconds

Year is 365.2422 days approximately.

The common year consists of 365 days.

The difference between a common year and a solar year is therefore 0.2422 of a day and we consider it by adding a whole day to every fourth year.

Consequently in every 4th year there are 366 days.

The years which have the extra day are called leap years. The day is inserted at the end of February, The difference between 4 common years and 4 solar years is 0.969 of a day.

If therefore, we add a whole day to every 4th year, we add too much by 0.0312 of a day. To take account of this, we omit the extra day three times every 400 years,

The thing is to ensure that each season may fall at the same time of the year in all years. In course of time, without these corrections, we should have winter in July and summer in January also .

With the very small variation, the present divisions of the year are those given in B.C 46 by Julius Caesar . The omission of the extra day three times in 400 years is called the Gregorian Correction. This correction was adopted

at once in 1582 in Roman Catholic Countries. but not in England until, 1752.
The Gregorian mode of reckoning is called the New Style, the former, the Old Style.
The New Style has not yet been adopted in Russia , so that they are now 13 days behind us as an Example What we call Oct. 26th they call 13th Oct . They have Christmas day on 7th of January and we have on 25th December every year.

In an ordinary year there are 365 days i.e., 52 weeks + 1 day
Therefore an ordinary years contains 1 odd day.
A leap year contains 2 odd days.

100 year = 76 ordinary years + 24 leap years.
= 76 odd days + 48 odd days
= 124 odd days = 17 weeks + 5 days. (in the consideration of weeks)
Therefore, 100 years contain 5 odd days.
200 years contains 3 odd days.
300 years contain 1 odd days

Since there are 5 odd days in 100 years, there will be 20 days in 400 years. But every 4th century is a leap year.
Therefore, 400 years contain 21 days. Here 400 years contain no odd days.

As First January 1 AD was Monday. we must count days from Sunday
i.e. Sunday for 0 odd days , Monday for 1 odd day , Tuesday for 2 odd days and so on.

Last day of a century cannot be either Tuesday. Thursday or Saturday.

The first day of a century must either be Monday. Tuesday, Thursday or Saturday.

Now let us observe the Examples

Ex How many times does the 29th days of the month occur in 400 consecutive years
1) 97 times 2) 4400 times 3) 4497 times 4) none
Sol: In 400 consecutive years there are 97 leap years. Hence in 400 consecutive years, February has the 29th day 97 times, and the remaining 11 months have the 29th day 400 x 11 or 4400 times.
Therefore, 29th day of the month occurs (4400 + 97) or 4497 times

Ex Given that on 10th November 1981 is Tuesday, what was the day on 10th November 1581
1) Monday 2) Thursday 3) Sunday 4) Tuesday
Sol: After every 400 years, the same day comes.
Thus if 10th November1981 was Tuesday, before 400 years i.e on 10th November 1581, it has to be Tuesday.

Exercise questions

1. What is the angle between the two hands of a clock when the time shown by the clock is 6.30 p.m. ?

a) 00
b) 50
c)30
d)150

Ans: option d
Explanation: q = 11/2 m – 30h
$\qquad$ = 11/2 *30 - 30 *6
$\qquad$ = /165-180/ = 150

2. At what time between 3 and 4 o'clock will the minute hand and the hour hand are   on the same straight line but facing opposite directions.

a) 3:49
b)3:15
c)3:39 $^1/_{11}$
d)3:49 $^1/_{11}$

Ans: option d
Explanation : On straight line means 180 degree angle.
180 = 11/2m- 30h
180 = 11/2 m –30*3
180 = 11/2m-90
(180+90)2 = 11m
m = 540/11 = 49 $^1/_{11}$

3. By how many degrees does the minute hand move in the same time, in which the hour hand move by 280 ?

a)168
b)336
c) 196
d) 376

Ans: option b
Explanation: 28*2 *6 = 336^0

4. At what time, between 3 o'clock and 4 o'clock, both the hour hand and minute hand coincide each other?

a) 3:30
b) 3:16 $^4/_{11}$
c) 3:16$^{11}/_4$
d) 3:16 $^7/_{11}$

Ans: Option b
Explanation : Coincide means 00 angle.
0 =11/2m –30*3
11m = 90*2 = 180
m= 180/11 = 16 $^4/_{11}$
So time = 3 : 16 $^4/_{11}$

5. How many degrees will the minute hand move, in the same time in which the second hand move 4800 ?
a) 60
b) 90
c) 40
d) 80

Ans: Option d
Explanation : Minute hand covers 480/60= 80

6. How many years have 29 days in February from 2001 to 2100.

a)26
b)25
c)23
d)24

Ans: option d
Explanation: 100th year is not a leap year. So 24 February's has 29 days

7. 2012 January 1st is Sunday, then which day is the Indian Independence day of the same year.

a) Saturday
b) Wednesday
c) Thursday
d) Friday

Ans: Option b
Explanation: 30+ 29+ 31 + 30 + 31 + 30 + 31+15 = 227/7 = reminder = 3
So Independence day is Wednesday

8. Which year has the same calendar as 1700 ?

a) 1705
b)1706
c)1707
d)1708

Ans: Option b

Explanation:

Year	:	1700	1701	1702	1703	1704	1705
Odd days	:	1	1	1	1	2	1

9. If Arun's birthday is on May 25 which is Monday and his sister's birthday is on July 13. Which day of the week is his sister's birthday?

a) Monday
b) Wednesday
c) Thursday
d) Friday

Ans: option a
Explanation: Reference day : May 25th Monday
Days from May 25th to July 13 = 6 + 30 +13 = 49
No of odd days : 49/7 = 0

10. March 1st is Wednesday. Which month of the same year starts with the same day?

a) October
b) November
c) December
d) None of these

Ans: Option b
Explanation:

Month	: Mar	April	May	June	July	Aug	Sept.	Oct.
Odd dys	: 3	2	3	2	3	3	2	3

Total 21 odd days. 21/7 = 0. So November has start with the same day

1. What is the angle between the two hands of a clock when the time shown by the

clock is 5.30 p.m. ?

a) 0°   b) 5°   c)3°   d)15°

2. By how many degrees does the minute hand move in the same time, in which the

   hour hand move by 18 degree ?

   a)168 degree   b)216 degree c) 196 degree d) 276 degree

3. A watch, which loses time uniformly, was observed to be 5 minutes fast at 8.00 p.m.
   on Thursday. It was noticed to be 7 minutes slow at 8.00 a.m. on the subsequent
   Monday. When did the watch show the correct time ?

   a) 7 a.m. Saturday   b) 7 a.m. on Friday

    c) 10a.m. on Sunday   d) 11 a.m. on Friday

4. In the time in which the second hand covers 3960 degrees, how many degrees does

   the hour hand move?

   a) 11   b) 5.5   c)3/4   d) 55

5. By how many degrees does the hour hand lag behind the minute hand in a span of

   42 minutes, if initially they are at the same position?

   a) 233° b)211 degree   c)231 degree   d)None of these

6. March 1st is Wednesday. Which month of the same year has the same calendar

   a)July   b) November   c) December   d)October

7. If today is Thursday , after 730 days which will be the day of the week ?

   a)Thursday   b)Friday   c)Saturday   d)Monday

8. A year starting with Monday and ending with Tuesday. How many days are there
   from 16th January to 15th March of that year.

   a) 58   b)60   c)59   d)61

9. The last day of a century cannot be :

   a) Monday   b) Wednesday c) Friday   d) Saturday

10. What will be the day of the week on 1st January, 2010 ?

    a) Friday   b) Saturday   c) Sunday   d) Monday

Answer & Explanations

 1. q = 11/2 m – 30h

      = 11/2 *30 - 30 *5

      = 165-150 = 15°

2. 18*2 *6 = 216 degree

3. The number of hours from 8:00 p.m. on Thursday to 8:00 a.m. on Monday = 84
   hours.

In 84 hours, the clock gained 12 minutes.

But to show the correct time, the clock has to gain 5 minutes.

∴. 5/12 *84 = 35 hours.

35 hours from 8:00 p.m. on Thursday is 7:00 a.m. on Saturday.

4. $3960/(360*2) = 5.5^\circ$

5. $42*6-(42/2) = 252-21 = 231$ degree

6. Odd days from March to October : $3 + 2 +3 +2 +3 +3 +2 +3 = 21/7 = 0$. If odd day is 0 then next month has the same calendar.

7. $730/7 = 2$ odd days. So Saturday will be the day of the week.

8. It is a leap year. So 16(Jan)+ 29(Feb) + 15(March) = 60 days

9. 100 years contains 5 odd days. So last day of 1st century is 'Friday'.

200 years contains $(5*2) =10 = 3$ odd days. So last day of second century is 'Wednesday'. 300 years contain 15 odd days = 1 odd day.

. : Last day of 3rd century is 'Monday'. 400 years contains 0 odd day.

: . Last day of 4th century is 'Sunday'.

Since the order is continually kept in successive cycles, we see that the last day of a century cannot be Tuesday, Thursday or Saturday

10. 2000 years have 2 odd days.

Number of odd days from 2001 −2009 = 11 odd days = 4 odd days. 1stJanuary 2010 has 1 odd day. Total number of odd days = (2 + 4 + 1) = 7 = 0 odd day

: . 1st January, 2010 will be a Sunday.