Logarithm

1.If x,a and m are any three numbers connected by the relation:

$m=a^x$ (a>0, a≠1), then,

"x" is defined as the logarithm of "m" to the base "a" and is written as:

$x= \log_a m$

2.Some important results:

(a) $m = a^{\log am}$

(b) $x = \log_a (a^x)$

(c) $\log_a 1 = 0$

3.Some important theorems:

(a) $\log_a (mn) = \log_a m + \log_a n$

(b) $\log_a (m/n) = \log_a m - \log_a n$

(c) $\log_a (m^n) = n. \log_a m$

(d) $\log_a m = (\log_b m) / (\log_b a)$ ……. Change of base theorem

(e) $\log_a a = 1$

(f) $\log_a b * \log_b a = 1$

Questions

1.If $a^x = b^y$, then
a.log a/b= x/y b.log a/ log b = x/y c.log a/ log b = y/x d.log b/a = x/y

2.2 $\log_{10} 5 + \log_{10} 8 - ½ \log_{10} 4$ = ?
a.2 b.4 c.2 + 2 $\log_{10} 2$ d.4 - 4 $\log_{10} 2$

3.$\log_a (ab) = x$, then $\log_b (ab)$ is :
a.1/x b.x/ (x+1) c.x/(1-x) d.x/(x-1)

4.If $\log_8 x + \log_8 1/6 = 1/3$, then the value of x is:
a.12 b.16 c.18 d.24

5.The value of ($\log_9 27 + \log_8 32$) is:
a.7/2 b.19/6 c.5/3 d.7

6.If $\log_{12} 27 = a$, then $\log_6 16$ is:
a.(3-a)/4(3+a) b.(3+a)/4(3-a) c.4(3+a)/(3-a) d.4(3-a)/(3+a)

7.The value of ($1/\log_3 60 + 1/\log_4 60 + 1/\log_5 60$) is:
a.0 b.1 c.5 d.60

8.If log x + log y = log (x+y), then,
a.x=y b.xy=1 c.y= (x-1)/x d.y=x/(x-1)

9.If log 27= 1.431, then the value of log 9 is:
a.0.934 b.0.945 c.0.954 d.0.958

10.If log 2= 0.030103, the number of digits in 2^{64} is :
a.18 b.19 c.20 d.21

Answer & Explanations

1.(c). $a^x = b^y$ => $\log a^x = \log b^y$ => x log a = y log b
=> log a/ log b = y/x

2.(a). $2 \log_{10} 5 + \log_{10} 8 - ½ \log_{10} 4$
$= \log_{10} (5^2) + \log_{10} 8 - \log_{10} (4^{1/2})$
$= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10} (25*8)/2$
$= \log_{10} 100 = 2$

3.(d). $\log_a (ab) = x$ => log b/ log a = x => (log a + log b)/ log a = x
1+ (log b/ log a) = x => log b/ log a = x-1
log a/ log b = 1/ (x-1) => 1+ (log a/ log b) = 1 + 1/ (x-1)
(log b/ log b) + (log a/ log b) = x/ (x-1) => (log b + log a)/ log b = x/ (x-1)
=>log (ab)/ log b = x/(x-1) => $\log_b (ab) = x/(x-1)$

4.(a). $\log_8 x + \log_8 (1/6) = 1/3$
=> (log x/ log 8) + (log $^1/_6$ / log 8) = log $(8^{1/3})$ = log 2
=> log x = log 2 – log 1/6 = log (2*6/1)= log 12

5.(c). Let $\log_9 27 = x$. Then, $9^x = 27$
=> $(3^2)^x = 3^3$ => 2x = 3 => x= 3/2
Let $\log_8 32 = y$. Then
$8^y = 32$ => $(2^3)^y = 2^5$ => 3y = 5 => y=5/3

6.(d). $\log_{12} 27 = a$ => log 27/ log 12 = a
=> log 3^3 / log (3 * 2^2) =a
=> 3 log 3 / log 3 + 2 log 2 = a => (log 3 + 2 log 2)/ 3 log 3 = 1/a
=> (log 3/ 3 log 3) + (2 log 2/ 3 log 3) = 1/3
=> (2 log 2)/ (3 log 3) = 1/a – 1/3 = (3-a)/ 3a
=> log 2/ log 3= (3-a)/3a => log 3 = (2a/3-a)log2
$\log_{16} 16$ = log 16/ log 6 = log 2^4/ log (2*3) = 4 log 2/ (log 2 + log 3)
= 4(3-a)/ (3+a)

7.(b). $\log_{60} 3 + \log_{60} 4 + \log_{60} 5 + \log_{60} (3*4*5)$
$= \log_{60} 60 = 1$

8.(d). log x + log y = log (x+y)
=> log (x+y) = log (xy)
=> x+y = xy => y(x-1) = x
=> y= x/(x-1)

9.(c). log 27 = 1.431 => log 3^3 = 1.431
=> 3 log 3= 1.431 => log 3 = 0.477
Therefore, log 9 = log 3^2 = 2 log 3 = (2*0.477) = 0.954

10.(c). log 2^{64} = 64 log 2 = (64*0.30103) = 1926592
Its characteristics is 19.
Hence, the number of digits in 2^{64} is 20.