Factorial: The continued product of first 'n' natural numbers is called the 'n factorial' and is denoted by n!. That is n!=1*2*3*4*.....*(n-1)*n

Eg: 4!=1*2*3*4

1. ${}^{n}P_{r}=n!/(n-r)!$
2. ${}^{n}P_{n}=n!$
3. ${}^{n}P_{1}=n$

1. ${}^{n}C_{r} = n!/(r! (n-r)!)$
2. ${}^{n}C_{1} = n$
3. ${}^{n}C_{0} = 1 = nCn$
4. ${}^{n}C_{r} = nCn-r$
5. ${}^{n}C_{r} = nPr/r!$

1.The number of all permutations of n distinct items or objects taken 'r' at a time is

$n(n-1)(n-2).....(n-(r-1)) = {}^{n}P_{r}$

2.The number of all permutations of n distinct objects taken all at a time is n!

3.The number of ways of selecting r items or objects from a group of n distinct items or objects is nCr = n!/(r! (n-r)!)

4. If there are n subjects of which p 1 are alike of one kind; p 2 are alike of another kind; p 3 are alike of third kind and so on and p r are alike of r thkind,

such that (p 1 + p 2 + ... p r) = n .

5.Then, number of permutations of these n objects= n! / (p1 !)(p2 !).....(pr !)

6.The number of all combinations of n things, taken r at a time is:

${}^{n}C_{r} = n!/(r! (n-r)!) = n(n-1)(n-2)/r!.....$ to r factors

7.The number of circular arrangements of n distinct items is (n – 1)! when there is a difference between clockwise and anticlockwise arrangements and (n – 1)!/2 when there is no difference between clockwise and anticlockwise arrangements.

8.If all possible n-digit numbers using n-distinct non-zero digits formed, the sum of all the numbers so formed is equal to (n – 1)! x [sum of the n digits] x {1111…..n times}.

Dividing given Items into groups:

1. The number of ways of dividing (p + q) items into two groups containing p and q items respectively is (p + q)!/p!q!

2. The number of ways of dividing 2p items into two equal groups of p each is (2p)! / (p!)2 when the two groups have distinct identity and (2p)!/2!(p!)2 when the two groups do not have distinct identity.

3. The number of ways of dividing (p + q + r) items into three groups containing p, q and r items respectively is (p + q + r)! / p!q!r!

4. The number of ways of dividing 3p items into three equal groups of p each is (3p)! / (p!)3 when the three groups have distinct identity and (3p)! / 3!(p!)3 when the three groups do not have distinct identity.

- ${}^{n}P_{r} = n!/ (n-r)!$
- ${}^{n}C_{r} = n!/r!(n-r)!$
- ${}^{n}C_{r} = nC(n-r)$
- ${}^{n}C_{r} = nPr/r!$

1. 0!=1
2. 1!=1
3. ${}^{n}P_{n}=n!$
4. ${}^{n}P_{1}=n$

5. ${}^{n}C_{1}=n$
6. ${}^{n}C_{n}=1$

1. Evaluate $\frac{50!}{47!}$

a. 102500

b. 112584

c. 117600

d. 118450

2. Find the value of ${}^{85}P_{3}$.

a. 565350

b. 595650

c. 535950

d. 565350

3. Find the value of $({}^{20}C_{18})*({}^{20}C_{20})$

a. 400

b. 380

c. 360

d. 350

4. How many words with or without meaning, can be formed by using all the letters of the word, 'ORANGE', using each letter exactly once?

a. 700

b. 720

c. 750

d. 800

5. There are 28 stations between Ernakulam and Chennai. How many second class tickets have to be printed, so that a passenger can travel from one station to any other station?

a. 800

b. 820

c. 850

d. 870

6. In how many ways can the letters of the word, 'TECHNOLOGY' be arranged?

a. 1804400

b. 1814400

c. 1714400

d. 1704400

7. A bag contains 2 yellow balls, 3 white balls and 5 red balls. In how many ways can two balls be drawn from the bag?

a. ${}^{2}C_{2}$

b. ${}^{10}C_{2}$

c. ${}^{8}C_{2}$

d. ${}^{5}C_{2}$

8. In how many ways can the letters of the word, 'LANGUAGE' be arranged in such a way that the vowels always come together?

a. 600

b. 700

c. 720

d. 750

9. In how many ways can the letters of the word, 'KEYBOARD' be arranged in such a way that the vowels always come together?

a. 4250

b. 4520

c. 4320

d. 4230

10. In how many ways can a team 16 be chosen out of a batch of 20 players?

a. 4845

b. 6852

c. 3125

d. 5846

11. How many ways can the letters of the word, 'MACHINE' be arranged so that the vowels may occupy only the odd positions?

a. 210

b. 576

c.144

d. 456

12. From a group of 5men and 4 women, 3 persons are to be selected to form a committee so that at least 2 men are there are on the committee. In how many ways can it be done?

a. 20

b. 50

c. 65

d. 86

13. In how many ways can a committee consisting of 4 men and 5 women be formed from a group of 7 men and 9 women?

a. ${}^{7}C_{4}\ {}^{9}C_{5}$

b. ${}^{4}C_{7}\ {}^{5}C_{9}$

c. ${}^{7}C5\ {}^{9}C_{4}$

d. ${}^{9}C_{4}\ {}^{7}C_{5}$

14. In how many ways can 5 boys and 3 girls sit around a table in such a way that no two girls sit together?

a. 1000

b. 1400

c.1440

d. 1800

Directions for questions 15 to 16: Refer the data below and answer the questions below:

A letter lock has 3 rings each containing 6 letters.

15. What is the maximum number of false trials that can be made before the lock is opened?

a. $3*{}^{26}C_{6}$

b. $({}^{26}C_{6})^{3}$

c. ${}^{26}C_{6}.\ 3!$

d. 215

16. How many such three letter passwords can exist?

a. 216

b. ${}^{26}C_{6}\ *3$

c. $({}^{26}C_{6})^{3}$

d. $({}^{26}C_{6})^{3}*6^{3}$

17. How many different words can be formed from the word DAUGHTER so that ending and beginning letters are consonants?

a. 7200

b. 14400

c. 360

d. 1440

18. Out of 6 consonants and 3 vowels, how many words of 4 consonants and 2 vowels can be formed?

a. 1050

b. 25200

c. 32400

d. 5800

19. A box contains 3 white balls, 4 black balls and 5 yellow balls. In how many ways can 4 balls be drawn from the box, if at least one yellow ball is to be included in the draw?

a. 652

b. 547

c.425

d. 356

20. In how many ways can 22 books on English and 20 books on Hindi be placed in a row on a shelf so that two books on Hindi may not be together?

a. 4586

b. 5896

c. 2415

d. 1771

Answer & Explanations

1. Evaluate $\frac{50!}{47!} = \frac{50*49*48*(47!)}{47!} = 50*49*48 = 117600$

2. ${}^{85}P_3 = \frac{85!}{(85-3)!} = \frac{85!}{82!} = \frac{85*84*83*82!}{82!} = 85*84*83 = 595650$

3. ${}^{20}C_{20} = 1$

$({}^{20}C_2)*({}^{20}C_{20}) = \frac{20!}{18!} * 1 = \frac{20*19*18!}{18!} = 20*19*1 = 380$

4. Exp: The word 'ORANGE' contains 6 different letters.

Therefore, Required number of words= Number of arrangement of 6 letters, taken all at a time

$= {}^6P_6 = 6! = 6*5*4*3*2*1 = 720$

5. Exp: The total number of stations= 30

From 30 Stations we have to choose any two stations and the direction of travel (Ernakulam to Chennai is different from Chennai to Ernakulam) in ${}^{30}P_2$ ways.

${}^{30}P_2 = 30*29 = 870$

6. Exp: The word 'TECHNOLOGY' contains 10 letters namely 2O, 1T, 1E, 1C, 1H, 1N, 1L, 1G, 1Y.

Therefore, Required number of ways= $\frac{10!}{(2!)(1!)(1!)(1!)(1!)(1!)(1!)(1!)(1!)} = \frac{10!}{2!}$

$= \frac{10*9*8*7*6*5*4*3*2*1}{2*1} = 1814400$

7. Exp: Total number of balls= 2+3+5 = 10

2 balls can be drawn from 10 balls in ${}^{10}C_2$

8. Exp: In the word 'LANGUAGE' we treat the vowels AUAE as one letter. Thus, we have LNGG (AUAE).

This we have 5 letters of which G occurs 2 times and the rest are different.

Number of ways arranging these letters= $\frac{5!}{2!} = 5*4*3 = 60$

Now, 4 letters of which A occurs 2 times and the rest are different, can be arranged in $\frac{4!}{} = 4*3 = 12$.

2!

Therefore, Required number of ways= 60*12= 720

9. Exp: In the word 'KEYBOARD' we treat the vowels EOA as one letter. Thus, we have KYBRD (EOA).

Thus we have 6 letters can be arranged in 6! = 720 ways

The vowels (EOA) can be arranged among themselves in 3! = 6 ways

Therefore, Required number of ways= (720*6)= 4320

10. Exp: Required number of ways= $^{20}C_{16} = {}^{20}C_{(20-16)} = {}^{20}C_4$

$$= \frac{20*19*18*17}{4*3*2*1} = 4845.$$

11. Exp: In the word 'MACHINE' 3 vowels and 4 consonants.

v v v v

— – — – — – —

Now, 3 vowels can be placed at any of 3 places, out of which 4 marked 1,3,5,7.

Number of ways arranging the vowels= $^4P_3$ = (4*3*2) =24

Also, 4 consonants at the remaining 4 positions may be arranged in = $^4P_4$= 4!= 24 ways.

Therefore, Required number of ways= (24*24) =576.

12. We have (2men and 1 woman) or (3men only)

Therefore, Required number of ways= ($^5C_2$*$^4C_1$) + ($^5C_3$)

$$= \frac{5*4}{2*1} * 4 + {}^5C_2$$

$$= 10*4 + 10$$

$$= 40 + 10 = 50$$

13. Exp: Group consisting of 7 men and 9 women

4 men can be selected from 7 men in $^7C_4$ ways

5 women can be selected from 9 women in $^9C_5$ ways

Therefore, Total number of ways= $^7C_4$ $^9C_5$

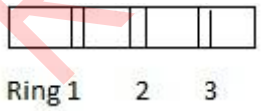
14. Exp: The 5 boys can be seated around a table in 4! Ways. In between them there are 5 places.

The 3 girls can be placed in the 5 places in $^5P_3$ ways.

Therefore, Required number of ways= 4!* $^5P_3$

=24*60=1440

15. Exp:



Maximum possible permutation of letters= 6*6*6= 216

Out of 216 different permutations only 1 is correct.

Maximum number of false trials= 216-1= 215

16. Exp: 1st ring: 6 out of 26 alphabets can be selected in $^{26}C_6$ ways.

And is for 2nd and 3rd ring.

Also, these 3 set of 6 letters can be arranged amongst themselves in 6^3ways.

Hence, total number of 3 letter passwords = ${}^{26}C_6 * {}^{26}C_6 * {}^{26}C_6 * 6^3$ ways.

17. Exp: Here total letters are 8,3 vowels and 5 consonants. Here 2 consonants can be chosen in ${}^5C_2$ ways and these 2 consonants can be put it in 2! Ways. The remaining 6 letters can be arranged in 6! Ways.

The words beginning and ending letters with consonant = ${}^5C_2 * 2! * 6! = 14400$

18. Number of ways of selecting (4 consonants out of 6) and (2 vowels out of 3)

$= {}^6C_4 * {}^3C_2$

$= {}^6C_2 * {}^3C_1$

$= \frac{6*5}{2*1} * 3$

$= 15*3 = 45$

Number of groups, each having 4 consonants and 2 vowels= 45.

Each group consists of 6 letters.

Number of ways of arranging 6 letters among themselves

$= 6! = (6*5*4*3*2*1) = 720$

Therefore, Required number of words= 45*720= 32400

19. Exp: We may have (1 yellow and 3 others) or (2 yellow and 2 others) or (3 yellow and 1 others) or (4 yellow).

Therefore, Required number of ways= $({}^4C_1 * {}^8C_3) + ({}^4C_2 * {}^8C_2) + ({}^4C_3 * {}^8C_1) + ({}^4C_4)$

$= 4 * \frac{8*7*6}{3*2*1} + \frac{4*3}{2*1} * \frac{8*7}{2*1} + ({}^4C_1 * 8) + 1$

$= 224+168+32+1 = 425.$

20. Exp: In order that two books on Hindi are never together, we must place all these books as under:

H E H E H E H….. H E H

Where H denotes the position of Hindi book and E that of English book.

Since there are 22 books on English, the number of places marked E are 23.

Now, 20 places out of 23 can be chosen in ${}^{23}C_{20} = {}^{23}C_3 = \frac{23*22*21}{3*2*1}$

$= 1771$ ways.

Hence the number of ways = 1771 ways

==========Recruitment16.in==========

Exercise questions

1. How many words can be formed by re-arranging the letters of the word ASCENT such that A and T occupy the first and last position respectively?
A)5!
B)4!
C)6! - 2!
D)6! / 2!

2. There are 2 brothers among a group of 20 persons. In how many ways can the group be arranged around a circle so that there is exactly one person between the two brothers?
A) 2 * 19!
B)18! * 18
C) 19! * 18
D)2 * 18!

3. There are 12 yes or no questions. How many ways can these be answered?
A) 1024

B) 2048
C) 4096
D)144

4. How many ways can 4 prizes be given away to 3 boys, if each boy is eligible for all the prizes?
A) 256
B) 12
C) 81
D) None of these

5. A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. In how many ways can they travel?
A) 9
B)26
C)126
D) 3920

6. How many numbers are there between 100 and 1000 such that at least one of their digits is 6?
A) 648
B) 258
C) 654
D)252

7. How many ways can 10 letters be posted in 5 post boxes, if each of the post boxes can take more than 10 letters?
A) 510
B) 105
C) 10P5
D) 10C5

8. In how many ways can the letters of the word EDUCATION be rearranged so that the relative position of the vowels and consonants remain the same as in the word EDUCATION?
A) 9!/4
B) 9!/(4!*5!)
C) 4!*5!
D) None of these

9. In how many ways can 15 people be seated around two round tables with seating capacities of 7 and 8 people?
A) 15!/(8!)
B) 7!*8!
C) (15C8)*6!*7!
D)2*(15C7)*6!*7!

10. If the letters of the word CHASM are rearranged to form 5 letter words such that none of the word repeat and the results arranged in ascending order as in a dictionary what is the rank of the word CHASM?
A) 24
B)31
C) 32
D)30

11. How many words of 4 consonants and 3 vowels can be made from 12 consonants and 4 vowels, if all the letters are different?
A) 16C7 * 7!
B) 12C4 * 4C3 * 7!
C) 12C3 * 4C4
D) 12C4 * 4C3

12. In how many ways can 5 letters be posted in 3 post boxes, if any number of letters can be posted in all of the three post boxes?
A) 5C3
B) 5P3
C) 53
D)35

13. How many number of times will the digit '7' be written when listing the integers from 1 to 1000?
A) 271
B) 300
C) 252
D)304

14. There are 6 boxes numbered 1, 2,...6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered.

The total number of ways in which this can be done is
A) 5
B) 21
C) 33
D) 60

15. What is the value of 1*1! + 2*2! + 3!*3! + ... n*n!, where n! means n factorial or n(n-A(n-2)...1
A) n(n-A(n-A!))
B) (n+A!)/(n(n-A))
C) (n+A! - n!) D) (n + A! - 1!)

Answers

1.B; 2.D; 3.C; 4.C; 5.C; 6.D; 7.A; 8.C; 9.C; 10.C; 11.B; 12.D; 13.B; 14.B; 15.D