Progression

Important Equations and Formula

Sum of first n natural numbers= n(n+1)/2
Sum of the squares of first n natural numbers= (n(n+1)(2n+1))/6
Sum of the cubes of first n natural numbers= $[n\ (n+1)/2]^2$
Sum of first n natural odd numbers= n^2
Average = Sum of items/ Number of items

Arithmetic Progression (AP): An AP is of the form a, a+d, a+2d, a+3d,..... where a is called the 'first term' and d is called the 'common difference'.
nth term of an AP; tn=a+(n-1)d
Sum of the first n terms of an AP; Sn= n/2 [2a+(n-1)d] or Sn= n/2 (first term+last term)

Geometrical Progression (GP):
A GP is of the form a, ar, ar^2, ar^3......... where a is called the 'first term' and r is called the 'common ratio'.
nth term of a GP; tn= ar^{n-1}
Sum of the first n terms in a GP; Sn= $a(1-r^n)/1-r$
Sum of infinite series of progression; S= a/(1-r)
Geometric mean of two number a and b is given as GM= sqrt(ab)

Harmonic Progression (HP)

If a_1,a_2,a_3,...................an are in AP, then $1/a_1$, $1/a_2$, $1/a_3$, ...$1/a_n$, are in HP
nth term of this HP, tn =$1/(1/a_1+(n-1)(a_1-a_2/a_1a_2)$) nth term of this HP from the end, tn=$1/\ (1/a_1-(n-1)(a_1-a_2/a_1a_2))$

If a and b are two non-zero numbers and H is harmonic mean of a and b then a, H, b from HP and then H=2ab/(a+b)

Arithmetico-Geometric series

A series having terms a, (a+d)r, $(a+2d)r^2$,...... etc is an Arithmetico-Geometric series where a is the first term, d is the common difference of the Arithmetic part of the series and r is the common ratio of the Geometric part of the series.

The nth term tn= $[a+(n-1)d]r^{n-1}$
The sum of the series to n terms is
Sn= $a/1-r+ dr\ (1-r^{n-1})/\ (1-r)^2$ - [a+(n-1)d]rn/ 1-r
The sum to infinity, S= $a/\ 1-r + dr\ (1-r^{n-1})/(1-r)^2$; r<1

Exponential Series

$e^x = 1+x/1!+x^2/2!+x^3/3!+$.......... ∞ (e is an irrational number)
coefficient of xn= 1/n!; Tn+1=$x^x/n!$
e^{-x} = 1-x/1!+x2/2!- x3/3!+.......... ∞

Logarithmic Series

$\log_e (1+x)= x-x^2/2+x^3/3+x^4/4+$........ ∞ $(-1<x \leq 1)$

$\log_e (1-x)= -x-x^2/2-x^3/3- x^4/4$ -........ ∞ (-1x< $\leq$ 1)

$\log_e (1+x)/(1-x)= 2-(x+x^3/3+x^5/5+$.........) ∞ $(-1<x \leq 1)$

1).
Find the 15^{th} term of an arithmetic progression whose first term is 2 and the common difference is 3.
a.45
b.38
c.44
d.40

2). Find the number of terms in an arithmetic progression with the first term 2 and the last term being 62, given that common difference is 2.
a.31
b.40

c.22
d.27

3). The first term of an arithmetic progression is 3 and the 10 th term is 21. Find 15 th and 22 nd terms.
a.21,35
b.31,45
c.30,46
d.29,40

4).
The 5 th term and 21 st term of a series in A.P are 10 and 42 respectively. Find the 31 st term.
a.50
b.55
c.65
d.62

5). Five times the fifth term of an A.P is equal to six times the sixth term of the A.P, What is the value of the eleventh term?
a.1
b.5
c.0
d.2

6). Find the sum of all 2 digit numbers divisible by 3.
a.2000
b.1665
c.1300
d.1448

7). The sum of n terms of a series in A.P is $6n^2 + 6n$. What is the 4 th term of the series?
a.38
b.49
c.60
d.48

8). How many terms are there in 2,4,8,16,..........1024?
a.10
b.6
c.9
d.8

9). The sum of the first five terms of a G.P
is 363. If the common ratio is 1/3 find the first term.
a.323
b.243
c.232
d.332

10). Find the sum of the following series,

1,4/5,16/25,64/125 ....∞

a.10
b.6
c.9
d.5

11). There are n arithmetic means between 3 and 45 such that the sum of these arithmetic means is 552. find the value of n.
a.11

b.15
c.17
d.23

12). Find the last term of a G.P whose first term is 9 and common ratio is (1/3) if the sum of the terms of the G.P is (40/3)
a.1/3
b.2/5
c.¼
d.2/3

13).
Find the common ratio of three numbers in G.P whose product is 216 and the sum of the products taken in pairs is 114.
a.2 or ½
b.2/3 or 3/2
c.¾ or 4/3
d.4 or ¼

14). In an A.P consisting of 27 terms, the sum of the first three terms is 21 and that of the three middle terms is 93. Find the first term and the common difference.
a.6,3
b.6,23
c.7,3
d.5,2

15). Find the first term and the common ratio of a G.P whose fourth term is 250 and seventh term is 31,250
a.2/5,25
b.4,5/2
c.1,16
d.2,5

16). The sum of the first eight terms of a geometric series is 10,001 times the sum of its four terms . Find the common ratio of these series
a. $\sqrt{1,000}$
b. $\sqrt{10}$
c.10
d.100

17). What is the sum of the first 15 terms of an A.P whose 11 th and 7 th terms are 5.25 and 3.25 respectively
a.56.25
b.60
c.52.5
d.None of these

18). If sum of three numbers in A.P is 33 and sum of their squares id 491, then what are the three numbers.
a.5,11,17
b.7,11,15
c.9,11,13
d.3,11,19

19). Four angles of a quadrilateral are in G.P. Whose common ratio is an intiger. Two of the angles are acute while the other two are obtuse. The measure of the smallest angle of the quadrilateral is
a.12
b.24
c.36

d.48

20).
The sum of the terms of an infinite G.P is 7 and the sum of the cubes of the terms is 1,225. Find the first term of the series.
a.35/3
b.35/2
c.15/2
d.9/4

Answer & Explanations

1. Exp. n th term of A.P = a +(n-1) *d
= 2+(15-1)*3 , = 2 + 42 = 44.

2. Exp. The n th term = a +(n-1) *d
62 = 2 +(n- 1) *2, 62 – 2 = (n-1) *2, n = 60/2 +1 = 31.

3. Exp. a = 3, T_{10} = a +9d =21, 3 +9d = 21, d =18/9 = 2
T_{15} = a + 14d =3 +14*2 = 31, T_{22} = a + 21d = 3+ 21*2 = 45

4. Exp. a + 4d = 10 ………….. (1)
a + 20d = 42 ………….. (2)
Eqn (2) – Eqn (1) gives 16d = 32 , d = 2
Substituting d =2 in either (1) or (2), a = 2.
31 st term = a + 30d = 2 + 30*2 = 62

5. Exp. 5(a + 4d) = 6 (a + 5d), 5a + 20d = 6a + 30d, a + 10d = 0,
i.e 11 th = 0

6. Exp. All 2 digit numbers divisible by 3 are 12,15,18,21,………. 99
This is an A.P with a = 12, and d = 3, Let it contain n terms
Then, 12 + (n – 1) *3 = 99, or n = (99-12) /3 + 1 = 30
Required sum = 30/2*(12 + 99) = 15*111 = 1665

7. Exp. 4 th term = sum of 4 terms - sum of 3 terms
= (6* 4^2 + 6*4) -(6* 3^2 +6*3), = (96+24) – (54+18)
= 120 - 72 = 48

8. Exp. Let the number of terms be n. Then,
2 * 2^{n-1} = 1024 , 2^{n-1} = 512 = 2^9
n – 1 = 9, n = 10

9. Exp. Sum of the first n terms of a G.P = $\frac{a(1 - r^n)}{1 - r}$,

$\frac{a[1 - (1/3)^5]}{1 - 1/3}$ = 363

a = $\frac{363[1-1/3]}{1-(1/3)^5}$ = (363* 2/3) / (1- 1/242) = 242/ (242/243)
a = 243

10. Exp. It is a G.P with infinite terms. The common ratio is (4/5) and this is less than 1. The sum to infinity of a series with first term 'a ' and common ratio 'r' =[a/(1-r)], The sum to infinity = 1/ (1 – 4/5) = 5

11. Exp. Arithmetic mean = 552/ n = 1/2(3 +45) , = 24
n = 552/24 =23

12. Exp. Sum of the G.P . = (First term - r * last term)/1 – r
40/3 = $\frac{9 - 1/3 (\text{last term})}{2/3}$

Last term = (-40/3*2/3+9) * 3 = - 80/3 + 27 = 1/3

13. Exp. Let the terms be a/r, a , ar
a/r *a * ar = a^3 =216, a =6
(a/r*a)+ (a*ar) + (ar*a/r) =114, a^2/r + a^2 r + a^2 =114
a^2 (1/r + r +1) = 114, 36[(1+ r^2 +r)/r] =114
6 [(1 +r^2+ r)/r]= 19, 6 r^2 -13 r + 6 = 0,
On solving, r = 2/3 or 3/2

14. Exp. As the sum of the first three terms will be thrice the second term, 3 (a + d) =21, also the sum of 13 th,14th and 15 th = 3 (a+ 13d) = 93
On solving the two eqns, d = 2, 3 (a+2) = 21, a = 5

15. Exp. a*r^3 = 250, a *r^6 =31,250, r^3 =31,250/250 = 125
r = 5, a* 125 = 250, a = 2

16. Exp. a (r^8 - 1)/(r - 1) = 10001 a (r^4 –1)/(r – 1)
r^4 +1 = 10001, r =10

17. Exp. a +10d = 5.25, a+6d = 3.25, 4d = 2, d = ½
a +5 = 5.25, a = 0.25 = ¼, s_{15} = 15/2 (2 * ¼ + 14 * ½)
= 15/2 (1/2 +14/2) = 15/2 *15/2 =225/ 4 = 56.25

18. Exp. a + (a + d) + (a + 2d) = 3(a + d) = 33,
a + d = 11, or second term = 11 , first term = 11– d ,
Then (11- d $)^2$ + 11^2 + (11 + d $)^2$ = 491
2 d^2 = 491 - (3 * 121) = 491 - 363 = 128
d^2 = 64, d = 8, a = 3,

19. Let the angles be a, ar, ar^2, ar 3.
Sum of the angles = a (r^4- 1) /r -1 = a (r^2+ 1) (r + 1) = 360
a< 90 , and ar< 90, Therefore, a (1 + r) < 180, or (r^2 + 1) > 2
Therefore, r is not equal to 1. Trying for r = 2 we get a = 24
Therefore, The angles are 24, 48, 96 and 192.

20. Exp. S a = a/ (1 – r) = 7 ………… (1)
Sum to infinity of the cubes = a^3 / 1 –r 3 = 1,225
From (1) a^3 / (1 – r $)^3$ = 7^3 = 343
Therefore, (1 – r $)^3$ / 1 – r^3 = 1225/343,
(1+ r^2 -2r)/ (1 + r^2 + r) = 25/7
7 + 7 r^2 –14r = 25 + 25 r + 25 r^2
18 r^2 +39r + 18 = 0, on solving r = -3/2 or -2/3
for an infinite G.P | r | < 1, r = - 2/3
Therefore, a / [1 – (- 2/3)] = 7, a = 7 * 5/3 = 35/3