I. **Laws of Indices:**

i. $a^m * a^n = a^{m+n}$

ii. $a^m/a^n = a^{m-n}$
iii. $(a^m)^n = a^{mn}$
iv. $(a^b)^n = a^n b^n$
v. $(a/b)^n = a^n/b^n$
vi. $a^0 = 1$

II. **Surds:** Let a be a rational number and n be a positive integer such that $a^{1/n} = \sqrt[n]{a}$ is irrational. Then, na is called a surd of order n.

III. **Laws of Surds:**

i. $\sqrt[n]{a} = a^{1/n}$

ii. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

iii. $\sqrt[n]{a/b} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

iv. $\left(\sqrt[n]{a}\right)^n = a$

v. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

vi. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

1. To find $\sqrt{(a + \sqrt{b})}$ write it in the form $m + n + 2\sqrt{mn}$, such that $m + n = a$ and $4mn = b$, then $\sqrt{(a + \sqrt{b})} = \pm(\sqrt{m} + \sqrt{n})$

2. $(\sqrt{a}.\sqrt{a}.\sqrt{a}....\infty) = a$
3. If $(\sqrt{a} + \sqrt{a} + \sqrt{a}.....\infty) = p$, then $p(p - 1) = a$.
4. If $a + \sqrt{b} = c + \sqrt{d}$, then $a = c$ and $b = d$.

Examples:
1. Simplify: (i) $(81)^{3/4}$ (ii) $(1/64)^{-5/6}$ (iii) $(256)^{-1/4}$

Solution: (i) $(81)^{3/4} = (3^4)^{3/4} = 3^3 = 27$.
(ii) $(1/64)^{-5/6} = 64^{5/6} = (2^6)^{5/6} = 2^5 = 32$

(iii) $(256)^{-1/4} = (1/256)^{1/4} = [(1/4)^4]^{1/4} = 1/4$

2. If $x=3+2\sqrt{2}$, then the value of $(\sqrt{x}- (1/\sqrt{x}))$ is:.....
Solution:

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$$

$$= (3 + 2\sqrt{2}) + \frac{1}{(3 + 2\sqrt{2})} - 2$$

$$= (3 + 2\sqrt{2}) + \frac{1}{(3 + 2\sqrt{2})} \times \frac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})} - 2$$

$$= (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) - 2$$

$$= 4.$$

$$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 2.$$

Questions

1. If *m* and *n* are whole numbers such that $m^n = 169$, then the value of $(m - 1)^{n+1}$ is:
a. 1
b. 13
c. 169
d. 1728
2. The simplified form of $\frac{x^{9/2} \cdot \sqrt{y^7}}{x^{7/2} \cdot \sqrt{y^3}}$ is:
a. x^2/y^2
b. $x^2 \cdot y^2$
c. xy
d. x^2/y

3. If $\sqrt{(3 + \sqrt[3]{x})} = 2$, then x is equal to :
a. 1
b. 2
c. 4
d. 8

4. If x is an integer, find the minimum value of x such that $0.00001154111 \times 10^x$ exceeds 1000.
a. 8
b. 1
c. 7
d. 6

5. Which among the following is the greatest?
a. 2^{3^2}
b. 2^{2^3}
c. 3^{2^3}
d. 3^{3^3}

6. Solve for *m* if $49(7^m) = 343^{3m+6}$
a. -8/6
b. -2
c. -4/6
d. -1

7. Solve for $2^{y^{\sqrt{2}^2}} = 729$.
a. ±3
b. ±1
c. ±2
d. ±4

8. $\sqrt{[200\sqrt{[200\sqrt{[200.....\infty]}]}}$ = ?
a. 200
b. 10
c. 1
d. 20

9. If a and b are positive numbers, $2^a = b^3$ and $b^a = 8$, find the value of a and b.
a. a = 2, b = 3
b. a = 3, b = 2
c. a = b = 3
d. a = b = 2

10. If $4^{4m+2} = 8^{6m-4}$, solve for *m*.
a. 7/4
b. 2
c. 4
d. 1

11. If $2^x \times 16^{2/5} = 2^{1/5}$, then x is equal to:
a. 2/5
b. -2/5
c. 7/5
d. -7/5

12. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals :
a. xz/x + z
b. xz/2(x + z)
c. xz/2(x – z)
d. 2xz/(x + z)

13. If $7^a = 16807$, then the value of $7^{(a-3)}$ is:
a. 49
b. 343
c. 2401
d. 10807

14. If $3^x - 3^{x-1} = 18$, then the value of x^x is:
a. 3
b. 8
c. 27
d. 216

15. If $2^{(x-y)} = 8$ and $2^{(x+y)} = 32$, then x is equal to:
a. 0
b. 2
c. 4
d. 6

16. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is:
a. 0
b. 1
c. 1/abc
d. abc

17. 125 x 125 x 125 x 125 x 125 = $5^?$
a. 5
b. 3
c. 15
d. 2

18. If $5^{2n-1} = 1/(125^{n-3})$, then the value of n is:

a. 3
b. 2
c. 0
d. -2

19. If $x = 5 + 2\sqrt{6}$, then $\frac{(x-1)}{\sqrt{x}}$ is equal to:

a. $\sqrt{2}$
b. $2\sqrt{2}$
c. $\sqrt{3}$
d. $2\sqrt{3}$

20. Number of prime factors in $\frac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}}$ is :

a. 56
b. 66
c. 112
d. None of these

Answer & Explanations

1. Exp: Clearly, m = 13 and n = 2.
Therefore, $(m-1)^{n+1} = (13-1)^3 = 12^3 = 1728$.
2. Exp: $\frac{x^{9/2} \cdot \sqrt{y^5}}{x^{7/2} \cdot \sqrt{y^3}}$ is: $= x^{(9/2-5/2)} \cdot y^{(7/2-3/2)} = x^2 \cdot y^2$
3. Exp: On squaring both sides, we get:
$3 + \sqrt[3]{x} = 4$ or $\sqrt[3]{x} = 1$.
Cubing both sides, we get $x = (1 \times 1 \times 1) = 1$
4. Exp: Considering from the left if the decimal point is shifted by 8 places to the right, the number becomes 1154.111. Therefore, $0.00001154111 \times 10^x$ exceeds 1000 when x has a minimum value of
8.
5. Exp: $2^{3^{\wedge}2} = 2^9$
$2^{2^{\wedge}3} = 2^8$
$3^{2^{\wedge}3} = 3^8$
$3^{3^{\wedge}3} = 3^{27}$
As $3^{27} > 3^8$, $2^9 > 2^8$ and $3^{27} > 2^9$. Hence 3^{27} is the greatest among the four.
6. Exp: $49(7^m) = 343^{3m+6} \Rightarrow 7^2 7^m \Rightarrow (7^3)^{3m+6} \Rightarrow 7^{2+m} = 7^{9m+18}$
Equating powers of 7 on both sides,
$m + 2 = 9m + 18$
$-16 = 8m \Rightarrow m = -2$.
7. Exp: $3^{y^{\wedge}\sqrt{2}^{\wedge}2} = 729$
$3^{y^{\wedge}2} = 3^4$ ($\sqrt{2}^2 = (2^{1/2})^2 = 2$)
equating powers of 2 on both sides,
$y^2 = 4 \Rightarrow y = \pm 2$
8. Exp: Let $\sqrt{[200\sqrt{[200\sqrt{[200\ldots\ldots..\infty]]]}}} = x$; Hence $\sqrt{200x} = x$
Squaring both sides $200x = x^2 \Rightarrow x(x - 200) = 0$
$\Rightarrow x = 0$ or $x - 200 = 0$ i.e. $x = 200$
As x cannot be 0, x = 200.
9. Exp: $2^a = b^3$ ….(1)
$b^a = 8$ …..(2)
cubing both sides of equation (2), $(b^a)^3 = 8^3$
$b^{3a} = (b^3)^a = 512$.
from (1), $(2^a)^a = (2^3)^3$.
comparing both sides, a = 3
substituting a in (1), b =2.
10. Exp: $4^{4m+2} = (2^3)^{6m-4} \Rightarrow 4^{4m+2} = 2^{18m-12}$
Equating powers of 2 both sides,
$4m + 2 = 18m - 12 \Rightarrow 14 = 14m \Rightarrow m = 1$.
11. Exp: $2^x \times 16^{2/5} = 2^{1/5}$
$\Rightarrow 2^x \times (2^4)^{2/5} = 2^{1/5} \Rightarrow 2^x \times 2^{8/5} = 2^{1/5}$.
$\Rightarrow 2^{(x+8/5)} = 2^{1/5}$

=> x + 8/5 = 1/5 => x = (1/5 – 8/5) = -7/5.

12. Exp: Let $a^x = b^y = c^z = k$. Then, $a = k^{1/x}$, $b = k^{1/y}$, $c = k^{1/z}$.
Therefore, $b^2 = ac \Rightarrow (k^{1/y})^2 = k^{1/x} \times k^{1/z} \Rightarrow k^{2/y} = k^{(1/x + 1/z)}$
Therefore, $2/y = (x + z)/xz \Rightarrow y/2 = xz/(x + z) \Rightarrow y = 2xz/(x + z)$.

13. Exp: $7^a = 16807, \Rightarrow 7^a = 7^5$, a = 5.
Therefore, $7^{(a-3)} = 7^{(5-3)} = 7^2 = 49$.

14. Exp: $3^x – 3^{x-1} = 18 \Rightarrow 3^{x-1}(3 – 1) = 18 \Rightarrow 3^{x-1} = 9 = 3^2 \Rightarrow x – 1 = 2 \Rightarrow x = 3$.

15. Exp: $2^{(x-y)} = 8 = 2^3 \Rightarrow x – y = 3$ ---(1)
$2^{(x+y)} = 32 = 2^5 \Rightarrow x + y = 5$ ---(2)
On solving (1) & (2), we get x= 4.

16. Exp: $a^1 = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz}$. Therefore, xyz = 1.

17. Exp: $125 \times 125 \times 125 \times 125 \times 125 = (5^3 \times 5^3 \times 5^3 \times 5^3 \times 5^3) = 5^{(3+3+3+3+3)} = 5^{15}$.

18. Exp: $5^{2n-1} = 1/(125^{n-3}) \Rightarrow 5^{2n-1} = 1/[(5^3)^{n-3}] = 1/[5^{(3n-9)}] = 5^{(9-3n)}$.
$\Rightarrow 2n – 1 = 9 – 3n \Rightarrow 5n = 10 \Rightarrow n = 2$.

19. Exp: $x = 5 + 2\sqrt{6} = 3 + 2 + 2\sqrt{6} = (\sqrt{3})^2 + (\sqrt{2})^2 + 2 \times \sqrt{3} \times \sqrt{2} = (\sqrt{3} + \sqrt{2})^2$
Also, $(x – 1) = 4 + 2\sqrt{6} = 2(2 + \sqrt{6}) = 2\sqrt{2}(\sqrt{2} + \sqrt{3})$.
Therefore, $\frac{(x-1)}{\sqrt{x}} = \frac{2\sqrt{2}(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})} = 2\sqrt{2}$.

20. Exp: $\frac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}} = \frac{(2 \times 3)^{12} \times (5 \times 7)^{28} \times (3 \times 5)^{16}}{(2 \times 7)^{12} \times (3 \times 7)^{11}} =$

$= \frac{2^{12} \times 3^{12} \times 5^{28} \times 7^{28} \times 3^{16} \times 5^{16}}{2^{12} \times 7^{12} \times 3^{11} \times 7^{11}} = 2^{(12-12)} \times 3^{(12+16-11)} \times 5^{(28+16)} \times 7^{(28-12-11)}$

$= 2^0 \times 3^{17} \times 5^{44} \times 7^{-5} = \frac{3^{17} \times 5^{44}}{7^5}$

Number of prime factors = 17 + 44 + 5 = 66.

==========Recruitment16.in==========

Exercise Questions

1.The value of $(\sqrt{8})^{1/3}$ is:
a.2
b. 4
c. 2
d. 8

Answer: Option c.
$(\sqrt{8})^{1/3} = (8^{1/2})^{1/3} = 8^{1/6} = (2^3)^{1/6} = 2^{1/2} = \sqrt{2}$.

2. The value of $5^{1/4} * (125)^{0.25}$ is:
a. $\sqrt{5}$
b.$5\sqrt{5}$
c.5
d.25
Answer: Option c
$5^{0.25} * (5^3)^{0.25} = 5^1 = 5$.

3. The value of $(32/243)^{-4/5}$ is:
a. 4/9
b. 9/4
c. 16/81
d. 81/16

Answer: Option d.
$(32/243)^{-4/5} = (243/32)^{4/5} = [(3/2)^5]^{4/5} = 81/16$

4. $(1/216)^{-2/3} \div (1/27)^{-4/3} = ?$
a. 3/4
b. 2/3
c. 4/9

d. 1/8
Answer: Option c.
$(1/216)^{-2/3} \div (1/27)^{-4/3} = 216^{2/3} \div 27^{4/3} = (63)^{2/3} \div (33)^{4/3} = 4/9$

5. $(2^{n+4} - 2.2^{n})/(2.2^{n+3}) = 2^{-3}$ is equal to:
a. 2^{n+1}
b. $-2^{n+1} + 1/8$
c. $9/8 - 2^{n}$
d. 1
Answer: Option d.

$(2^{n+4} - 2.2^{n})/2.2^{n+3} + 1/2^{3} = 7/8 + 1/8 = 1$

6. If $5\sqrt{5} * 5^{3} \div 5^{-3/2} = 5^{a+2}$, the value of a is:
a. 4
b. 5
c.6
d. 8

Answer: Option a
$5^{3/2} * 5^{3} \div 5^{-3/2} = 5^{a+2}$

$5^{3/2} + 3 + 3/2 = 5^{a+2}$
$3/2 + 3 + 3/2 = a+2$
$a+2=6; a=4$

7. If $\sqrt{2n} =64$, then the value of n is:
a. 2
b. 4
c. 6
d. 12

Answer: Option d
$\sqrt{2n} =64 \Rightarrow 2^{n/2} = 64= 2^{6}$
$n/2=6; n=12$

8.The simplified form of $(x^{7/2} /x^{5/2}).\sqrt{y3} /\sqrt{y}$)is :
a.x^{2}/y
b. x^{3}/y^{2}
c. x^{6}/y^{3}
d. xy
Answer: Option d

$(x^{7/2}/x^{5/2}). (\sqrt{y3} /\sqrt{y}) = x^{7/2 - 5/2}. y^{3/2 - 1/2} = xy$